ortogonal

ex 5:

$W = \langle \mathbb{R}^3 \{(1,2,3)\} \rangle$ $= \langle \mathbb{R}^3 \{(-2,1,0),(-3,-6,5)\} \rangle$

$W^\perp = \{(x,y,z) : \langle (x,y,z),(1,2,3) \rangle = 0\} =$

$= \{(x,y,z) : x + 2y + 3z = 0\} = \{(x,y,z) : x = -2y - 3z\} =$

$= \{(-2y-3z, y, z) : y, z \in \mathbb{R}\} = \langle \mathbb{R}^3 \{(-2,1,0),(-3,0,1)\} \rangle$

$\{(1,2,3),(-2,1,0),(-3,-6,5)\}$

$\text{Proj}_y x = \frac{\langle x,y \rangle}{\langle y,y \rangle} y$

$u_1 = (-2,1,0)$ $u_2 = (-3,0,1)$ $u_1^* = (-2,1,0)$

$u_2^* = (-3,0,1) - \frac{\langle (-3,0,1),(-2,1,0) \rangle}{\langle (-2,1,0),(-2,1,0) \rangle} (-2,1,0) = (-3,0,1) - \frac{6}{5}(-2,1,0) = (-\frac{3}{5}, -\frac{6}{5}, 1)$

$(-3,-6,5)$

ex 6:

Se G é simétrica ($G = G^T$) e se $\forall x \neq 0 : x^T G x > 0$

os valores próprios de G são todos > 0

então $\langle x,y \rangle := x^T G y$ é um produto interno

$(A^T A)^T = A^T (A^T)^T = A^T A$ ✓

$x \neq 0 : x^T (A^T A) x = (Ax)^T (Ax) = \langle Ax, Ax \rangle = \|Ax\|^2 > 0$ (pq. $Ax \neq 0$)

$\|Ax\| > 0 \Leftrightarrow Ax \neq 0 \Rightarrow x \in \text{Nuc}(A) \Rightarrow \boxed{x = 0}$

arc