Sinais

$u(n)-u(n-4)$ - de 0 a 3

$u(t)-u(t-4)$ - de 0 a 4

Sistemas

ex: $y(n)=n x(n)$

→ SM

→ C

$y(0)=0$

$y(1)=x(1)$

→ $\bar{I}$

$x(n)\rightarrow y(n)=n x(n)$

$x_1(n)=x(n-n_0)\rightarrow y_1(n)=n x_1(n)=n x(n-n_0)$

$\overset{?}{=}$ $\neq$

$y(n-n_0)=(n-n_0)x(n-n_0)$

→ L

$x_1(n)\rightarrow y_1(n)=n x_1(n)$

$x_2(n)\rightarrow y_2(n)=n x_2(n)$

$x_3(n)=\alpha x_1(n)+\beta x_2(n)\rightarrow y_3(n)=\alpha n x_1(n)+\beta n x_2(n)$

$\overset{?}{=}$ $=$

$\alpha y_1(n)+\beta y_2(n)=\alpha n x_1(n)+\beta n x_2(n)$

→ $\bar{E}$

Hip: $|x(n)|\leq \Delta x$

$|y(n)|=|n x(n)|=|n|.|x(n)|\leq n\,\Delta x$

$x(n)=1\rightarrow y(n)=n$

→ $\overline{\text{Invert}}$

$y(n)=n x(n)\rightarrow x(n)=\frac{y(n)}{n}$

$\downarrow$

$y(0)=0\ \forall x(0)$

série geométrica

$$\sum_{k=k_1}^{k_2} r^k = r^{k_1}\frac{1-r^{k_2-k_1+1}}{1-r}$$

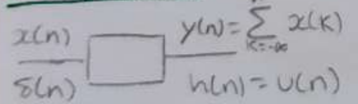
Convolução

$$y(n)=x(n)*h(n)=\sum_{k=-\infty}^{+\infty}x(k)h(n-k)=\sum_{k=-\infty}^{+\infty}h(k)x(n-k)$$

$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{+\infty}h(\tau)x(t-\tau)d\tau$$

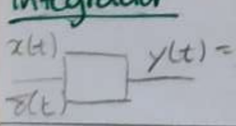
$\delta(t)$ → significa réplica

Acumulador

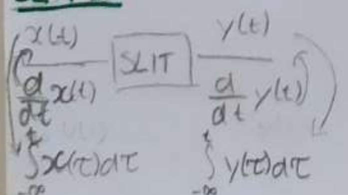


Integrador

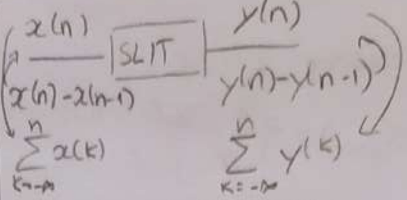
$$y(t)=\int_{-\infty}^{t}x(\tau)d\tau$$



SLIT's



↙ ex. 6 MAP ano passado



$u(n-1)=u(n)-\delta(n)$

Props SLIT's:

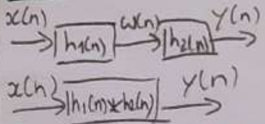
- sem memória → $h(t)=K.\delta(t)$
- causalidade → $h(t)=0, t<0$

✓ invariância temporal

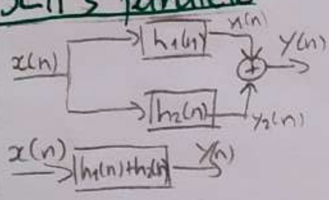
✓ linearidade

- estabilidade → $\int_{-\infty}^{+\infty}|h(t)|dt<\infty$
- invertibilidade → $\exists h_I, h*h_I=\delta$

SLIT's série



SLIT's paralelo



MAP30 22/23

③ $y(n)=\frac{1}{2-\sum_{k=-\infty}^{n}x(k)}$

substituir $x(k)$ pelas opções fazer limite de $y(n)$ $n\rightarrow\infty$ explode se $\lim=\infty$

Transformada Laplace

$$X(s)=\int_{-\infty}^{+\infty}x(t)e^{-st}dt, RC$$

$$\boxed{X(s)=\frac{\text{zeros}}{\text{polos}}}$$

polos - x

zeros - o

Inversa

$s=\sigma+j\omega$

$$x(t)=\frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega}X(s)e^{st}ds$$

$$H(s)=\frac{Y(s)}{X(s)}$$

Causalidade ↙

SLIT causal ⇒ RC associada a $H(s)$ é um ✗ semi-plano direito

SLIT causal ⇐ $H(s)$ racional e RC é um semi-plano direito

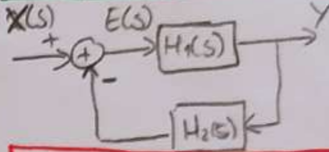
Estabilidade ↙

Um SLIT é estável sse RC conter eixo imaginário

SLIT causal c/ função de transferência racional é estável se todos os polos tiverem parte real negativa (semi-plano complexo esquerdo)

nº zeros não pode exceder nº polos

Realimentação SLIT's



$$\boxed{H(s)=\frac{H_1(s)}{1+H_1(s)H_2(s)}}$$

Transformada Fourier

$$X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$$

Inversa

$$\boxed{x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega}$$

$$\frac{e^{jk\pi t}+e^{-jk\pi t}}{2}=\cos(k\pi t)$$

$$\frac{e^{jk\pi t}-e^{-jk\pi t}}{2j}=\sin(k\pi t)$$

Série Fourier

$x(t)$ periódico com $T_0=\frac{2\pi}{\omega_0}$

$$\tilde{x}(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$$

$$\tilde{x}(t)=a_0+2\sum_{k=1}^{\infty}A_k\cos(k\omega_0 t+\theta_k)$$

Coeficientes:

$$a_k=\frac{1}{T_0}\int_{T_0}x(t)e^{-jk\omega_0 t}dt$$

$a_k=A_k e^{j\theta k}$ $a_{-k}=a_k^*$

$a_k^*=A_k e^{-j\theta k}$

$$y(t)=\sum_{k=-\infty}^{\infty}\underbrace{a_k H(jk\omega_0)}_{b_k}e^{jk\omega_0 t}$$

$x(t)=\sin(2\pi t)+\cos(3\pi t)$

$\omega_0=\text{mdc}(\omega_{01},\omega_{02})=\pi$

ransferência
odos os polos
…{sem-plano
der nº polos

s)

r

dt

$\boxed{{}^{\omega t}d\omega}$

$k\pi t)$

$n(k\pi t)$

$o = \frac{2\pi}{\omega_0}$

$(k\omega_0 t + \theta_k)$

$\omega_0 t \; dt$

$= a_k^*$

$e^{jk\omega_0 t}$

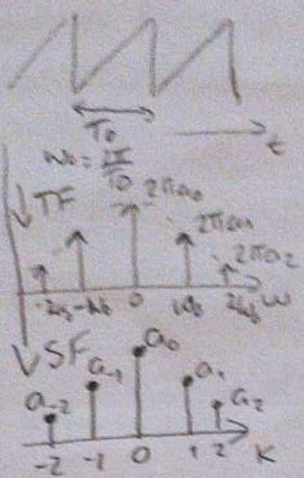
$3\pi t)$

T

prop dualidade

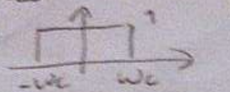
$$x(t) \xleftrightarrow{TF} X(j\omega) = f(\omega)$$

$$f(t) \xleftrightarrow{TF} F(j\omega) = 2\pi x(\omega)$$

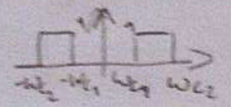


Filtros

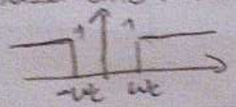
Passa-baixo ideal



Passa-banda ideal



Passa-alto ideal



Transformada Fourier discreta

$$x(n) = e^{j\Omega n} \rightarrow y(n) = H(e^{j\Omega})e^{j\Omega n}$$
$$n \in \mathbb{Z}, \; \Omega \in \mathbb{R}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\Omega n}$$

é sempre periódica em Ω c/ período 2π

Inversa

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$$

ex:

$$H(e^{j\Omega}) = \frac{2}{\underbrace{1 - \frac{3}{4}e^{-j\Omega} + \frac{1}{8}e^{-2j\Omega}}}$$

$e^{j\Omega} \rightarrow z$

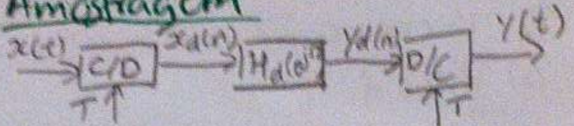
$$1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} = \underset{z^{-1} \cdot z^{-1}}{z^{-2}}\underbrace{\left(z^2 - \frac{3}{4}z + \frac{1}{8}\right)}_{(z-\frac{1}{4})(z-\frac{1}{2})} =$$

$$= \left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{2}z^{-1}\right)$$

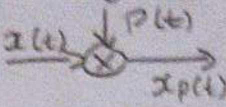
$z \rightarrow e^{j\Omega}$

$$\left(1 - \frac{1}{4}e^{-j\Omega}\right)\left(1 - \frac{1}{2}e^{-j\Omega}\right)$$

Amostragem



Amostragem por trem impulsos



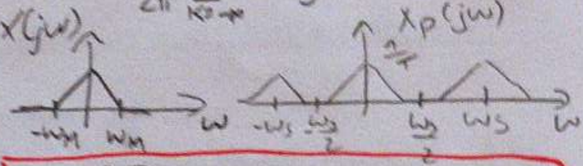
$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$x_p(t) = x(t)p(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$

T - período amostragem

$\updownarrow$ TF $\quad \omega_s = \frac{2\pi}{T}$ - frequência amostragem

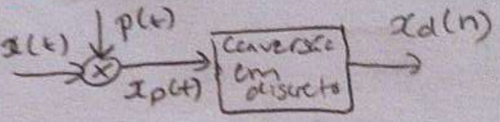
$$X_p(j\omega) = \frac{1}{2\pi}X(j\omega) * P(j\omega) =$$
$$= \frac{1}{2\pi}\sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$



$$\boxed{\omega_s = \frac{2\pi}{T} > 2\omega_M \quad \bar{n} \text{ tem aliasing}}$$

$\omega_s = \frac{2\pi}{T} < 2\omega_M$ tem aliasing

$$\boxed{\frac{\omega_s}{2} \rightarrow \text{frequência de Nyquist}}$$



$$x_d(n) = x(nT)$$

$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{+\infty} x(nT)e^{-j\Omega n}$$

$$X_p(j\omega) = \sum_{n=-\infty}^{+\infty} x(nT)e^{-j\omega Tn}$$

$\Omega = \omega T \Leftrightarrow \omega = \frac{\Omega}{T}$ $\quad \boxed{\Omega = 2\pi\frac{\omega}{\omega_s}}$

$$\boxed{X_d(e^{j\Omega}) = X_p\left(j\frac{\Omega}{T}\right)}$$

P1 1º exame 22/23

a) Polos: 2 {

zero: 1 {

RC: Re(s) >

c) Apesar de ter eixo imaginário tem + zeros q polos faz c/ que SLIT comporte parcialmente como diferenciador produzindo saída infinita quando entrada tem descontinuidades

entrada limitada, mas descontínua temos $y(0)=\infty$

d) que torna eq diferencial

$\frac{7+2s}{s^2+4s+3}$ $\begin{matrix}\rightarrow \text{entrada} \\ \rightarrow \text{saída}\end{matrix}$

Q5 2º exame 22/23

representa a resposta ao escalão unitário
↓
u(t)

A=3 B=2

$\lim_{t\to\infty} y(t) = B \overset{TVF}{\Leftrightarrow} \lim_{s\to0} sY(s) = B \Leftrightarrow$

$\Leftrightarrow \lim_{s\to0} H(s) = 2 \Leftrightarrow H(0)=2$

$x(t)=u(t) \Leftrightarrow X(s)=\frac{1}{s}$

$Y(s)=X(s)H(s) \Leftrightarrow Y(s) = \frac{H(s)}{s} \Leftrightarrow sY(s)=H(s)$

$\lim_{t\to0} y(t) = A \overset{TVI}{\Leftrightarrow} \lim_{s\to\infty} sY(s) = A \Leftrightarrow \lim_{s\to\infty} H(s) = A \Leftrightarrow$

$\Leftrightarrow \lim_{s\to\infty} H(s) = 3$

$\lim_{s\to\infty} \frac{3s^2+12s+12}{s^2+5s+6} \overset{*}{=} \lim_{s\to\infty} \frac{3s^2}{s^2} = 3$

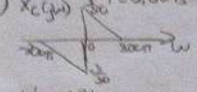
$H(0) = \frac{3\times0+12\times0+12}{0+5\times0+6} = \frac{12}{6} = 2$

P2 2º exame 22/23

a) $x_c(t)$ real? ver simetria $X_c(j\omega)$ e justificar c/ tabelas

t=0 → usar TF inversa

b) $X_c(j\omega)$ $\frac{3}{50}$ T=0,004s

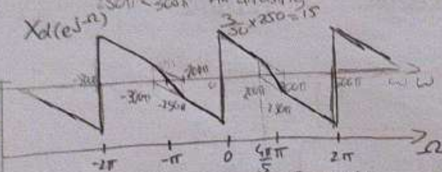


$H_d(e^{j\Omega}) = 1$

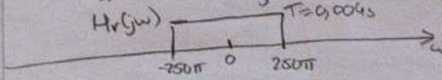
$\omega_s = \frac{2\pi}{T} = 500\pi$ rad/s

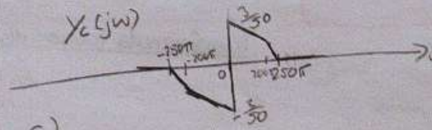
freq. Nyquist → $\frac{\omega_s}{2} = 250\pi$ rad/s

$250\pi < 300\pi$ há aliasing

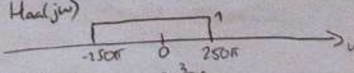


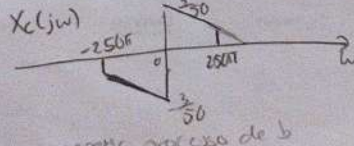
$\Omega = 2\pi\frac{\omega}{\omega_s}$



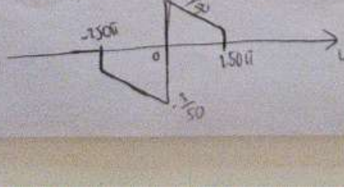


c)





repetir processo de b

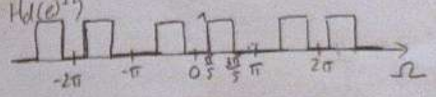


P2 1º exame 22/23

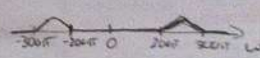
d) $H_c(j\omega) = \begin{cases} 1, & 50\pi \leq |\omega| \leq 150\pi \\ 0, & c.c. \end{cases}$

$H_d(e^{j\Omega}) = H_c\left(j\frac{\Omega}{T}\right), \Omega \in [-\pi, \pi[$ + réplicas 2π-periódicas

$\Omega = 2\pi\frac{\omega}{\omega_s}$



sem antialiasing $|X_c(j\omega)-Y_c(j\omega)|$



com anti-aliasing $|X_c(j\omega)-Y_c(j\omega)|$